In 1952, Harry Markowitz introduced the Modern Portfolio Theory to finance. Since then, we got the idea of optimization. In general, MPT states that investors should maximize portfolio expected return for a given amount risk, which is a convex optimization problem in the mathematics field.

During this summer, I added three optimization engines to PortfolioAnalytics(PA) package to provide more precise solutions in shorter running time. Next, I will introduce these three methods from lower dimension objective to higher dimension objective, then the complex constraints situation.

The basic optimization is linear programming problems, which can be used to max desired reward over conditional value at risk. The excepted shortfall is the mean of loss greater than certain threshold, usually 5% quantile of historical return. Even though it seems like a statistic of historical data, we can transform it into a linear equation form which combined VaR and upper limitation of VaR:

In above equation, is number of observations, aka the length of historical data; is the weight vector; and is the historical data matrix. Since we introduced new variables to optimization equation, we need to set constraints to this new variable. Assume vector . Then we know and . The constraints of weight vector are determined by portfolio constraints. Then in the general linear equation, we have which , and constraint matrix A according to portfolio constraints. I did a comparison test on the new Rglpk and other existed engine, and Rglpk provided an excellent result:

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| --- | --- | --- |
| method | Running time (s) | Mean Over ES |
| DEoptim | 9.218 | 0.091 |
| random | 63.49 | 0.064 |
| pso | 23.85 | 0.056 |
| GenSA | 47.66 | 0.089 |
| Rglpk | 0.11 | 0.182 |

The test sample is a historical monthly return data of 12 CTA over past 6 years. And we can see that Rlgpk can find a asset combination which provide the best MoE score which means we can get more return given same risk. Also, the running time of Rglpk is shortest among five engines.

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| --- | --- | --- |
|  | Sharpe Ratio (%) | Running Time (s) |
| DEoptim | 19.90 | 14.826 |
| random | 13.94 | 61.983 |
| pso | 19.55 | 24.187 |
| GenSA | 13.00 | 42.768 |
| osqp | 23.89 | 0.019 |

If we regard the volatility of a portfolio as risk, then the question becomes a quadratic programming problem. The quadratic programming problems have a quadratic form objective. The essential of this kind of problem is finding subject to . By using quartile recursion method on osqp engine, the PA package can solve the mean over volatility optimization in a short time now. The comparison test result as following:

Since the Sharpe Ratio optimization is the most common one in the portfolio management, the Sharpe Ratio from every engine is close to each other. However, we can see that osqp is the fastest one, it requires three less orders of magnitude. If we consider the high frequency optimization, this will save us lots of time.

Even though the above two engines can solve the most common objectives, the real-world condition is rather than complex, since every portfolio manager has their own constraints. Therefore, I introduced mco engine into PA package to solve multi criteria optimization problems. It supports customized objectives such as median, return and quantile of return. Also, it can optimize according to various constraints which can not be use in other engines, such as position limitation, turn over limitation, etc. More important, although it supports highly customized optimization problem, the running time of mco is controlled in a desire level. The comparison test result as following:

|  |  |  |
| --- | --- | --- |
| method | Running time (s) | Mean Over ES |
| DEoptim | 16.498 | 0.045 |
| random | 126.61 | 0.125 |
| pso | 70.31 | 0.065 |
| GenSA | 137.13 | 0.061 |
| mco | 0.62 | 0.159 |

In this test serial, the constraint includes position limitation, turn over limitation. Since we added more complex constraints, we can find that the running time raised one order of magnitude. However, the mco provided the best MoE score in the shortest time.